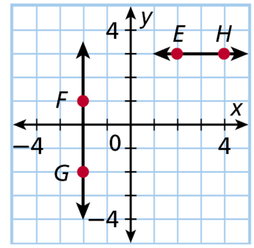
Pre-AP Geometry Study Guide 12-1 Lines that intersect Circles (pp 792-800)

Are You Ready Chapter 12 Pretest & skills.

<u>Attendance Problems.</u> Write the equation of each item.

1. $\overleftrightarrow{FG}$ **2.** $\overleftrightarrow{EH}$



<u>Solve.</u>

3. 2(25 - x) = x + 2 **4.** 3x + 8 = 4x

- I can identify tangents, secants, and chords.
- I can use properties of tangents to solve problems.

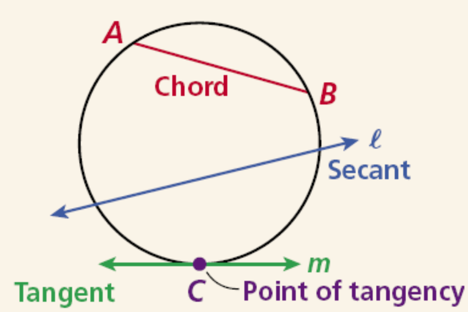
"Transform adversity into an enjoyable challenge!"—*Author and Researcher, Mihalyi Czikszent*

Vocabulary	
Interior of a circle	Concentric circles
Exterior of a circle	Tangent circles
Chord	Common tangent
Secant	Tangent of a circle
Point of tangency	Congruent circles

Common Core

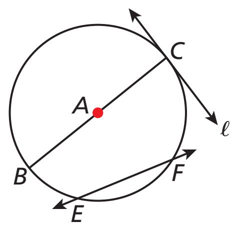
- **CC.9-12.G.C.2** Identify and describe relationships among inscribed angles, radii, and chords.
- **CC.9-12.G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **CC.9-12.G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

1 Identifying Lines and Segments That Intersect Circles

Identify each line or segment that intersects ⊙A.



chords: $\overline{EF}$ and $\overline{BC}$

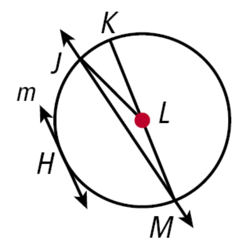
tangent: ℓ

radii: $\overline{AC}$ and $\overline{AB}$

secant: $\overleftrightarrow{EF}$

diameter: $\overline{BC}$

<u>Example 1</u>. Identify each line or segment that intersects ○L..



A. Chord

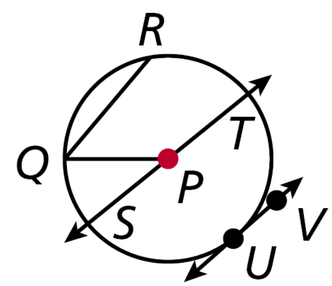
B. Secant

B. Tangent

C. Diameter

D. Radius

9. <u>Guided Practice.</u> Identify each line or segment that intersects $\odot P$.



A. Chord

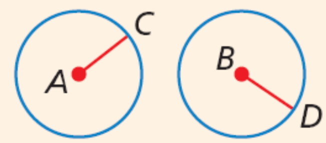
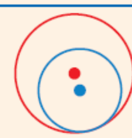
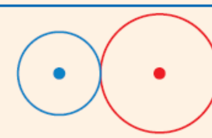
B. Secant

B. Tangent

C. Diameter

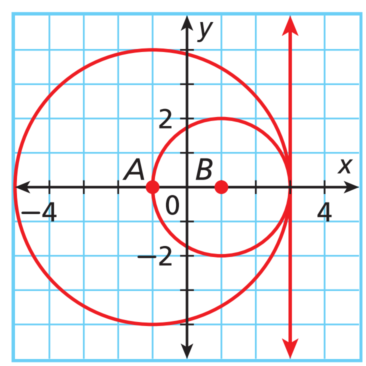
D. Radius

Pairs of Circles

TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ if $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ if $\odot A \cong \odot B$.
Concentric circles are coplanar circles with the same center.	
Two coplanar circles that intersect at exactly one point are called **tangent circles**.	 Internally tangent circles  Externally tangent circles

2 Identifying Tangents of Circles

Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



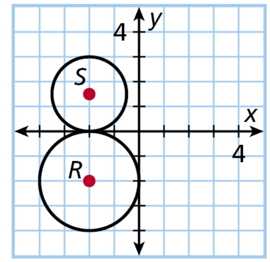
radius of $\odot A$: 4 — *Center is $(-1, 0)$. Pt. on $\odot$ is $(3, 0)$. Dist. between the 2 pts. is 4.*

radius of $\odot B$: 2 — *Center is $(1, 0)$. Pt. on $\odot$ is $(3, 0)$. Dist. between the 2 pts. is 2.*

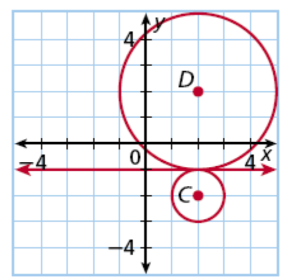
point of tangency: $(3, 0)$ — *Pt. where the $\odot$s and tangent line intersect*

equation of tangent line: $x = 3$ — *Vert. line through $(3, 0)$*

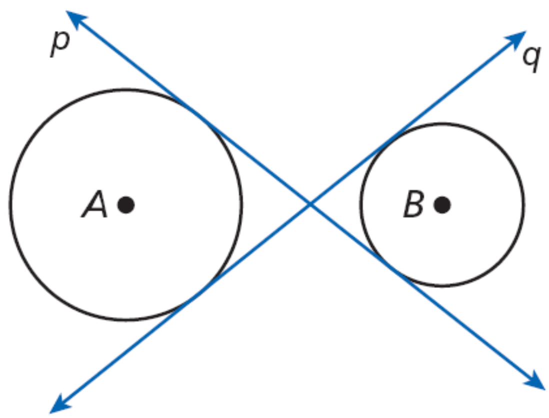
Example 2. Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



10. Guided Practice. Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

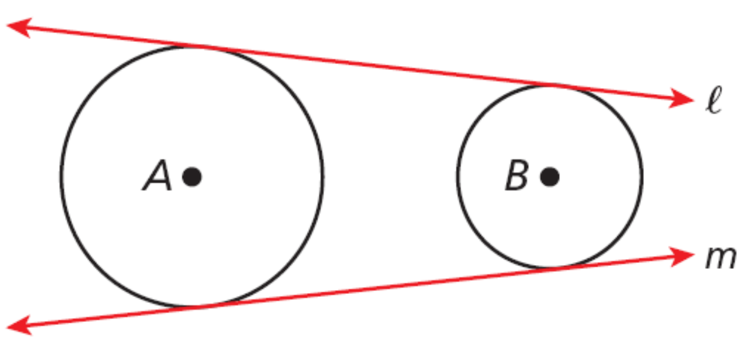


A **common tangent** is a line that is tangent to two circles.



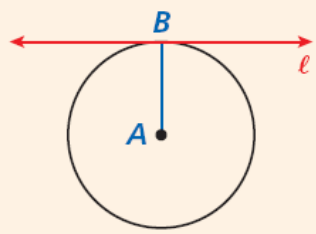
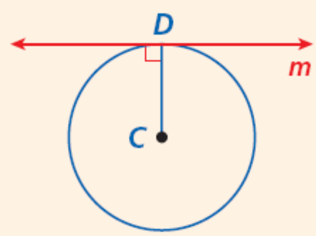
Lines ***p*** and ***q*** are common internal tangents to ⊙*A* and ⊙*B*.

A **common tangent** is a line that is tangent to two circles.

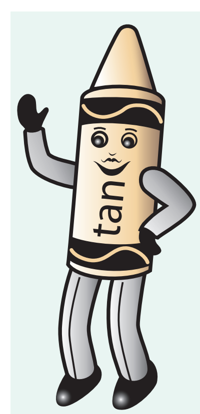


Lines ***ℓ*** and ***m*** are common external tangents to ⊙*A* and ⊙*B*.

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-1-1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (line tangent to ⊙ → line ⊥ to radius)	 ℓ is tangent to ⊙A	$\ell \perp \overline{AB}$
11-1-2 If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle. (line ⊥ to radius → line tangent to ⊙)	 m is ⊥ to $\overline{CD}$ at D	m is tangent to ⊙C.

Question: What math term is shown in the picture?
Answer: tangent



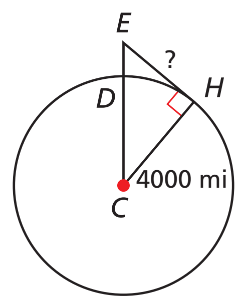
3 Problem Solving Application

The summit of Mount Everest is approximately 29,000 ft above sea level. What is the distance from the summit to the horizon to the nearest mile?

1 Understand the Problem

The **answer** will be the length of an imaginary segment from the summit of Mount Everest to Earth's horizon.

2 Make a Plan



Draw a sketch. Let C be the center of Earth, E be the summit of Mount Everest, and H be a point on the horizon. You need to find the length of $\overline{EH}$, which is tangent to $\odot C$ at H. By Theorem 12-1-1, $\overline{EH} \perp \overline{CH}$. So $\triangle CHE$ is a right triangle.

3 Solve

$ED = 29{,}000$ ft	*Given*
$= \dfrac{29{,}000}{5280} \approx 5.49$ mi	*Change ft to mi.*
$EC = CD + ED$	*Seg. Add. Post.*
$= 4000 + 5.49 = 4005.49$ mi	*Substitute 4000 for CD and 5.49 for ED.*
$EC^2 = EH^2 + CH^2$	*Pyth. Thm.*
$4005.49^2 = EH^2 + 4000^2$	*Substitute the given values.*
$43{,}950.14 \approx EH^2$	*Subtract 4000^2 from both sides.*
$210 \text{ mi} \approx EH$	*Take the square root of both sides.*

4 Look Back

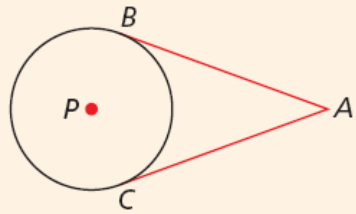
The problem asks for the distance to the nearest mile. Check if your answer is reasonable by using the Pythagorean Theorem. Is $210^2 + 4000^2 \approx 4005^2$? Yes, $16{,}044{,}100 \approx 16{,}040{,}025$.

Example 3. Early in its flight, the Apollo 11 spacecraft orbited Earth at an altitude of 120 miles. What was the distance from the spacecraft to Earth's horizon rounded to the nearest mile?

11. <u>Guided Practice</u>. A highway patrol officer is using laser radar on the side of the road. If it is a flat road, how far do you have to drive before you are out of range of range of the radar?

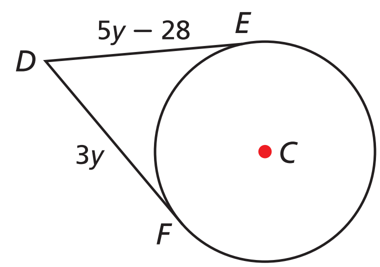
12-1 Lines that Intersect a Circle: (p 798) 11-15.

Theorem 11-1-3

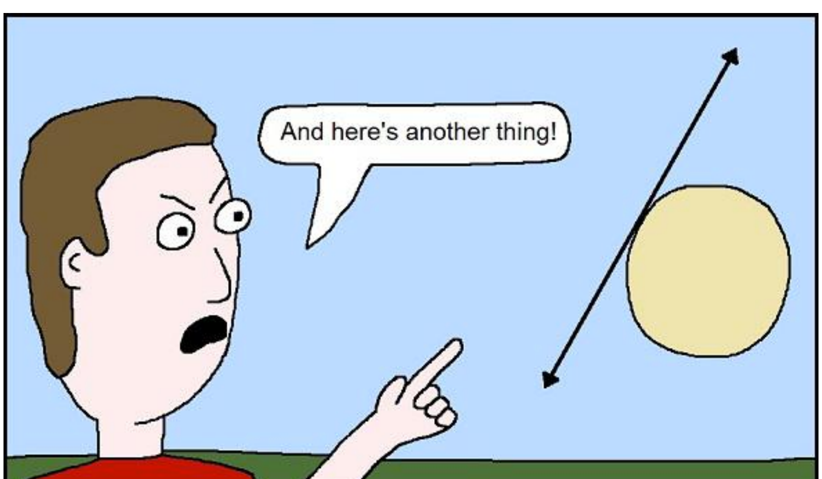
THEOREM	HYPOTHESIS	CONCLUSION
If two segments are tangent to a circle from the same external point, then the segments are congruent. (2 segs. tangent to ⊙ from same ext. pt. → segs. ≅)	 $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot P$.	$\overline{AB} \cong \overline{AC}$

4 Using Properties of Tangents

$\overline{DE}$ and $\overline{DF}$ are tangent to $\odot C$. Find DF.

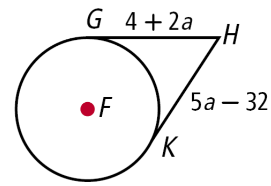


$DE = DF$	*2 segs. tangent to ⊙ from same ext. pt. → segs. ≅.*
$5y - 28 = 3y$	*Substitute 5y − 28 for DE and 3y for DF.*
$2y - 28 = 0$	*Subtract 3y from both sides.*
$2y = 28$	*Add 28 to both sides.*
$y = 14$	*Divide both sides by 2.*
$DF = 3(14)$	*Substitute 14 for y.*
$= 42$	*Simplify.*



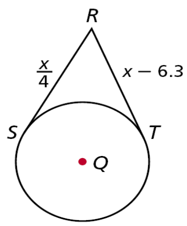
Steve was always going off on a tangent.

Example 4. $\overline{HK}$ & $\overline{HG}$ are tangent to $\odot F$. Find HG.

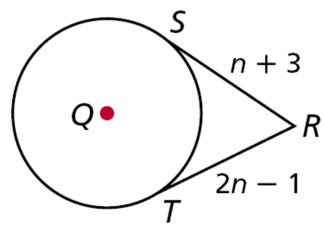


<u>Guided Practice</u>. $\overline{RS}$ & $\overline{RT}$ **are tangent to** $\bigcirc Q$. **Find RS.**

12.



13.



12-1 Lines that Intersect a Circle: (p 798) 11-17, 20, 26.